

5.1 Exploring What Makes Triangles Congruent



Resource Locker

Essential Question: How can you show that two triangles are congruent?

Explore Transforming Triangles with Congruent Corresponding Parts

You can apply what you've learned about corresponding parts of congruent figures to write the following true statement about triangles.

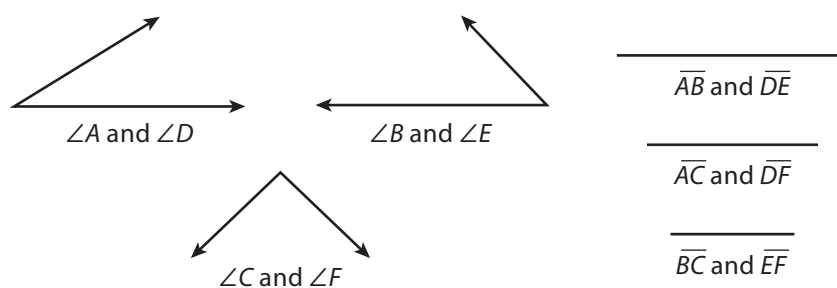
If two triangles are congruent, then the corresponding parts of the triangles are congruent.

The statement is sometimes referred to as *CPCTC*. The converse of CPCTC can be stated as follows.

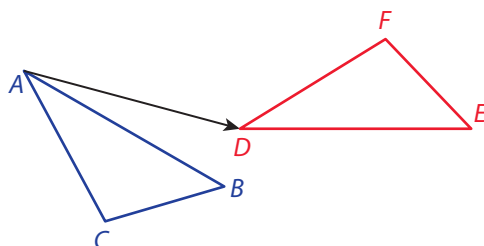
If all corresponding parts of two triangles are congruent, then the triangles are congruent.

Use a straightedge and tracing paper to explore this converse statement.

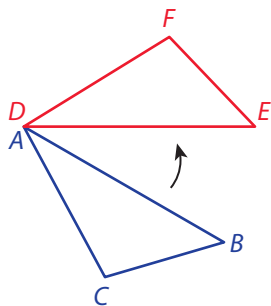
- A** Trace the angles and segments shown to draw $\triangle ABC$. Repeat the process to draw $\triangle DEF$ on a separate piece of tracing paper. Label the triangles.



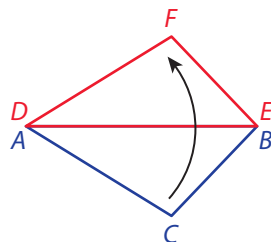
- B** What must you do to show that the triangles are congruent?
- C** Flip the piece of tracing paper with $\triangle ABC$ and arrange the two triangles on a desk as shown in the figure. Then move the tracing paper with $\triangle ABC$ so that point A maps to point D . Name the rigid motion that you used.



- D** Name a rigid motion you can use to map point B to point E . How can you be sure the image of B is E ?



- E** Name a rigid motion you can use to map point C to point F .



- F** To show that the image of point C is point F , complete the following.

$\angle A$ is reflected across \overleftrightarrow{DE} , so the measure of the angle is preserved. Since $\angle A \cong \angle D$, you can conclude that the image of \overrightarrow{AC} lies on ?. It is given that $\overline{AC} \cong$?, so the image of point C must be ?.

- G** What sequence of rigid motions maps $\triangle ABC$ onto $\triangle DEF$?

Reflect

- Discussion** Is there another sequence of rigid motions that maps $\triangle ABC$ onto $\triangle DEF$? Explain.
- Discussion** Is the converse of CPCTC always true when you apply it to triangles? Explain why or why not based on the results of the Explore.

Explain 1 Deciding If Triangles are Congruent by Comparing Corresponding Parts

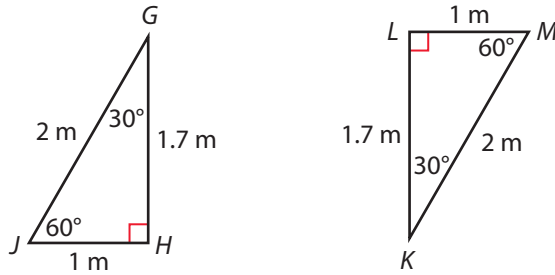
A **biconditional** is a statement that can be written in the form “ p if and only if q .” You can combine what you learned in the Explore with the fact that corresponding parts of congruent triangles are congruent to write the following true biconditional.

Two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.

To decide whether two triangles are congruent, you can compare the corresponding parts. If they are congruent, the triangles are congruent. If any of the corresponding parts are not congruent, then the triangles are not congruent.

Example 1 Determine whether the given triangles are congruent. Explain.

A



Compare corresponding sides to decide if they are congruent.

$GH = KL = 1.7$ m, $HJ = LM = 1$ m, and $GJ = KM = 2$ m.

So, $\overline{GH} \cong \overline{KL}$, $\overline{HJ} \cong \overline{LM}$, and $\overline{GJ} \cong \overline{KM}$.

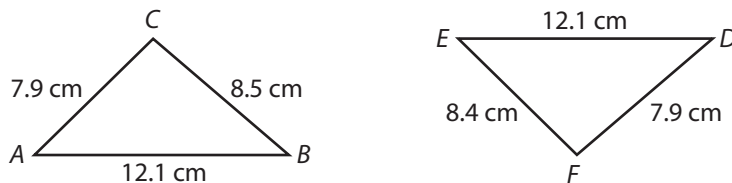
Compare corresponding angles to decide if they are congruent.

$m\angle G = m\angle K = 30^\circ$, $m\angle H = m\angle L = 90^\circ$, and $m\angle J = m\angle M = 60^\circ$.

So, $\angle G \cong \angle K$, $\angle H \cong \angle L$, and $\angle J \cong \angle M$.

$\triangle GHJ \cong \triangle KLM$ because all pairs of corresponding parts are congruent.

B



Compare corresponding sides to decide if they are congruent.

$AB = \boxed{DE} = \boxed{12.1}$ cm, so $\overline{AB} \cong \overline{DE}$. $AC = \boxed{DF} = \boxed{7.9}$ cm, so $\overline{AC} \cong \overline{DF}$.

However, $BC \neq \boxed{EF}$, so \overline{BC} is not congruent to \overline{EF} .

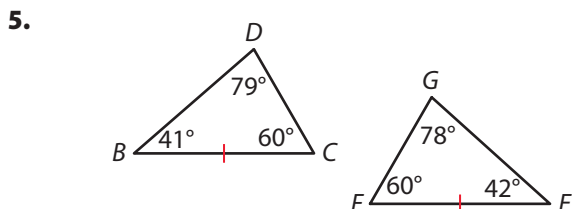
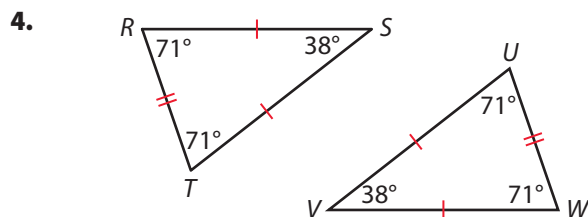
The triangles are not congruent because there is a pair of corresponding sides that are not congruent.

Reflect

- Critique Reasoning** The **contrapositive** of a conditional statement “if p , then q ” is the statement “If not q , then not p .” The contrapositive of a true statement is always true. Janelle says that you can justify Part B using the contrapositive of CPCTC. Is this accurate? Explain your reasoning.

Your Turn

Determine whether the given triangles are congruent. Explain your reasoning.

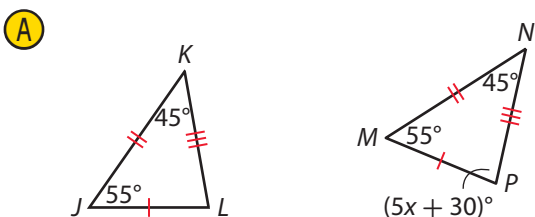


Explain 2 Applying Properties of Congruent Triangles

Triangles are part of many interesting designs. You can ensure that triangles are congruent by making corresponding sides congruent and corresponding angles congruent. To do this, you may have to use the Triangle Sum Theorem, which states that the sum of the measures of the angles of a triangle is 180° . You will explore this theorem in more detail later in this course.



Example 2 Find the value of the variable that results in congruent triangles.



Step 1 Identify corresponding angles.

$\angle M$ corresponds to $\angle J$, because they have the same measure and they are formed by congruent corresponding sides. Similarly, $\angle N$ corresponds to $\angle K$. So, $\angle P$ corresponds to $\angle L$.

Step 2 Find $m\angle L$.

Triangle Sum Theorem $m\angle J + m\angle K + m\angle L = 180^\circ$

Substitute. $55^\circ + 45^\circ + m\angle L = 180^\circ$

Simplify. $100^\circ + m\angle L = 180^\circ$

Subtract 100° from each side. $m\angle L = 80^\circ$

Step 3 Write an equation to find the value of x .

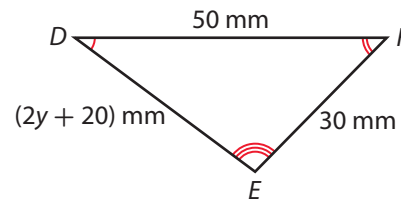
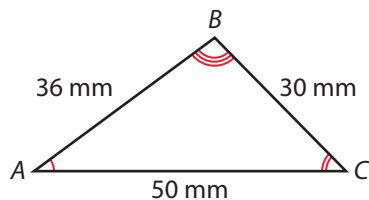
Set corresponding measures equal. $m\angle P = m\angle L$

Substitute. $5x + 30 = 80$

Subtract 30 from each side. $5x = 50$

Divide each side by 5. $x = 10$

B



Step 1 Identify corresponding sides, beginning with side \overline{DE} .

$\angle A \cong \angle D$, $\angle B \cong \angle E$, and $\angle C \cong \angle F$, so \overline{DE} corresponds to \overline{AB} .

Step 2 Write an equation to find the value of y .

Set corresponding measures equal. $DE = 36$ mm

Substitute. $2y + 20 = 36$

Subtract 20 from each side. $2y = 16$

Divide each side by 2. $y = 8$

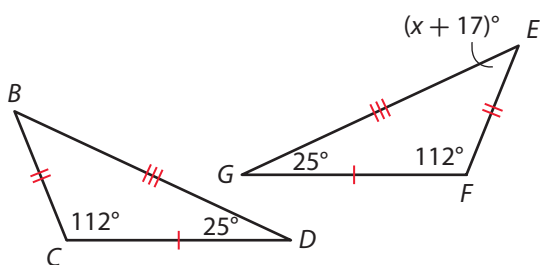
Reflect

6. The measures of two angles of $\triangle QRS$ are 18° and 84° . The measures of two angles of $\triangle TUV$ are 18° and 76° . Is it possible for the triangles to be congruent? Explain.

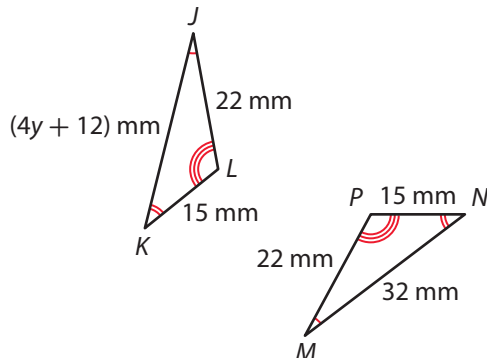
Your Turn

Find the value of the variable that results in congruent triangles.

7.



8.



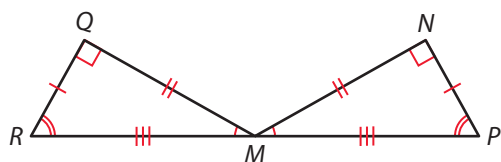
Elaborate

- All three angles of $\triangle ABC$ measure 60° and all three sides are 4 inches long. All three angles of $\triangle PQR$ measure 60° and all three sides are 4 inches long. Can you conclude that the triangles are congruent? Why or why not?
- Use the concept of rigid motion to explain why two triangles cannot be congruent if any pair of corresponding parts is not congruent.
- Essential Question Check-In** $\triangle PQR$ and $\triangle STU$ have six pairs of congruent corresponding parts and $\triangle PQR$ can be mapped onto $\triangle STU$ by a translation followed by a rotation. How are the triangles related? Explain your reasoning.

Evaluate: Homework and Practice



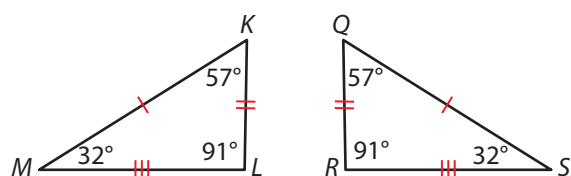
- Describe a sequence of rigid motions that maps $\triangle MNP$ onto $\triangle MQR$ to show that $\triangle MNP \cong \triangle MQR$.



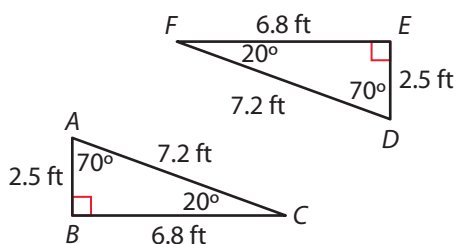
- Online Homework
- Hints and Help
- Extra Practice

For 2-5, determine whether the given triangles are congruent. Explain your reasoning.

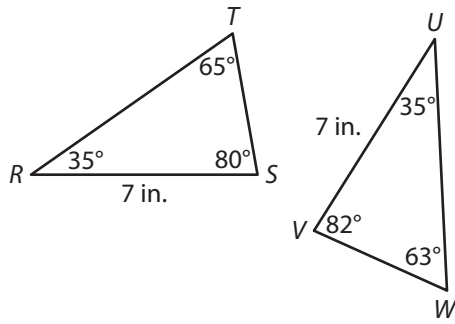
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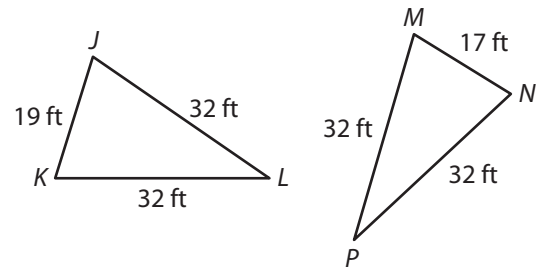
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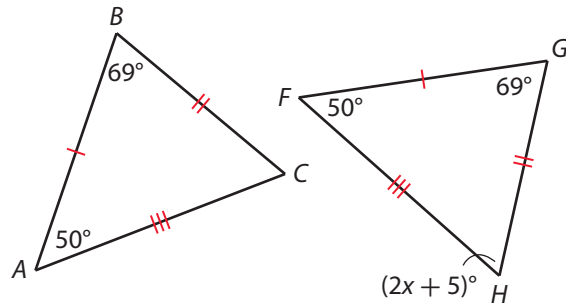


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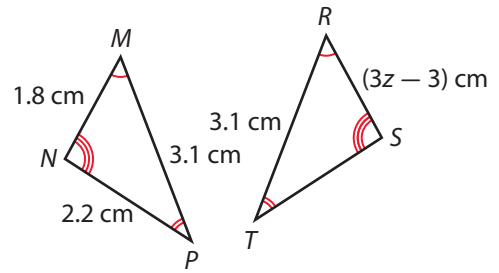


Find the value of the variable that results in congruent triangles.

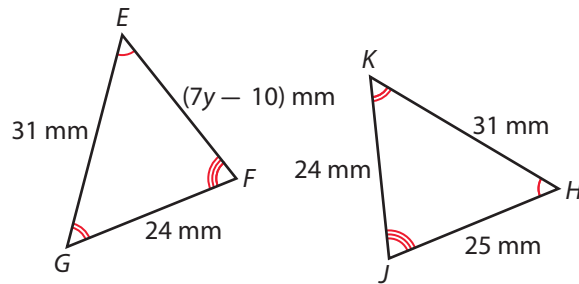
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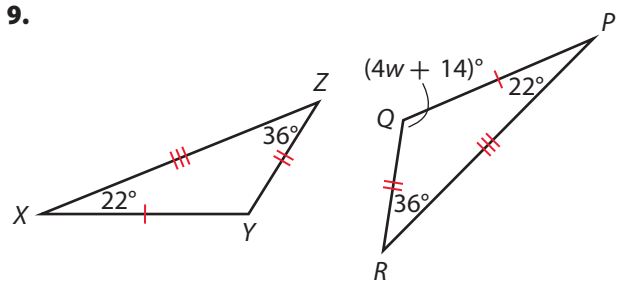
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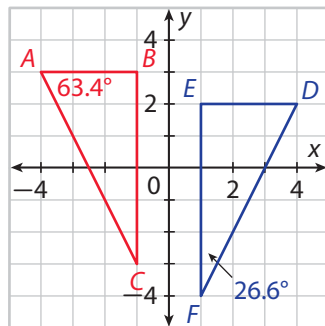


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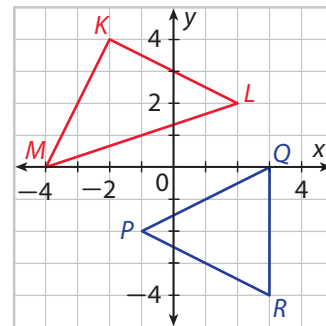


For 10-13, determine whether the given triangles are congruent. Explain.

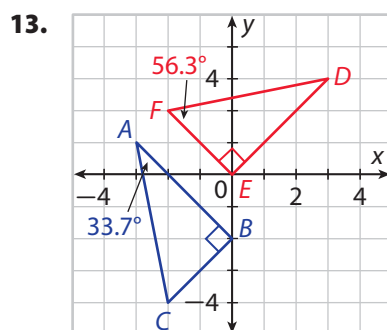
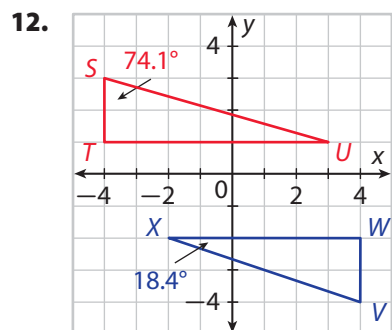
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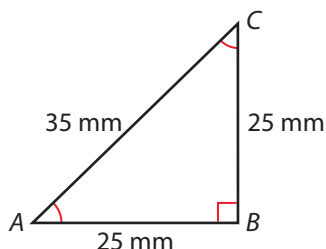
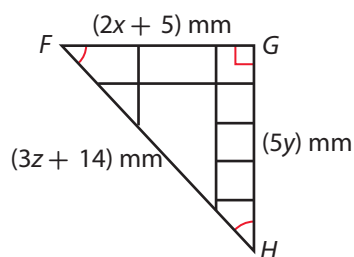
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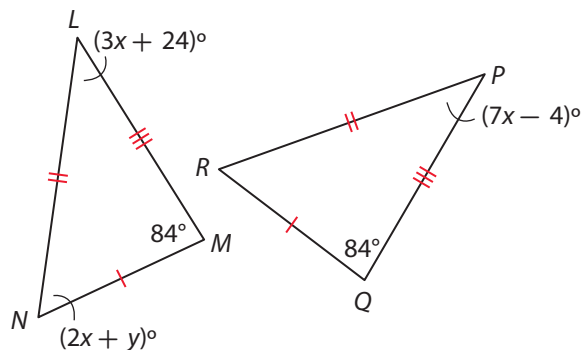
Determine whether the given triangles are congruent. Explain.



14. $\triangle FGH$ represents an artist's initial work on a design for a new postage stamp. What must be the values of x , y , and z in order for the artist's stamp to be congruent to $\triangle ABC$?



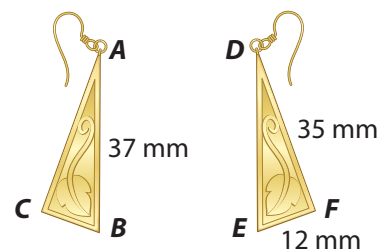
15. **Multi-Step** Find the values of the variables that result in congruent triangles.



Determine whether each statement is always, sometimes, or never true. Explain your reasoning.

16. If $\triangle ABC$ has angles that measure 10° and 40° , and $\triangle DEF$ has angles that measure 40° and 120° , then $\triangle ABC \cong \triangle DEF$.
17. Two triangles with different perimeters are congruent.
18. If $\triangle JKL \cong \triangle MNP$, then $m\angle L = m\angle N$.
19. Two triangles that each contain a right angle are congruent.

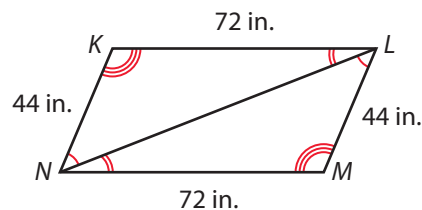
- 20.** Tenaya designed the earrings shown. She wants to be sure they are congruent. She knows that the three pairs of corresponding angles are congruent. What additional measurements should she make? Explain.



- 21.** Determine whether $\triangle JKL$ and $\triangle PQR$ are congruent or not congruent based on the given information.
- $m\angle J = m\angle K = m\angle L = 60^\circ$, $m\angle P = m\angle Q = m\angle R = 60^\circ$, $JK = KL = JL = 1.2$ cm, $PQ = QR = PR = 1.5$ cm
 - $m\angle J = 48^\circ$, $m\angle K = 93^\circ$, $m\angle P = 48^\circ$, $m\angle R = 39^\circ$, $\overline{JK} \cong \overline{PQ}$, $\overline{KL} \cong \overline{QR}$, $\overline{JL} \cong \overline{PR}$
 - $\angle J \cong \angle P$, $\angle K \cong \angle Q$, $\angle L \cong \angle R$, $JK = PQ = 22$ in., $KL = QR = 34$ in., $JL = PR = 28$ in.
 - $m\angle J = 51^\circ$, $m\angle K = 77^\circ$, $m\angle P = 51^\circ$, $m\angle R = 53^\circ$
 - $m\angle J = 45^\circ$, $m\angle K = 80^\circ$, $m\angle Q = 80^\circ$, $m\angle R = 55^\circ$, $JK = PQ = 1.5$ mm, $KL = QR = 1.3$ mm, $JL = PR = 1.8$ mm

H.O.T. Focus on Higher Order Thinking

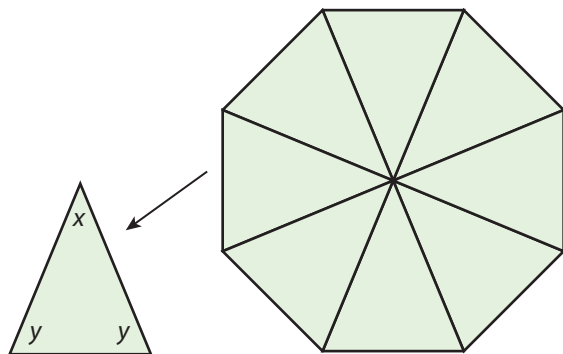
- 22. Counterexamples** Isaiah says it is not necessary to check all six pairs of congruent corresponding parts to decide whether two triangles are congruent. He says that it is enough to check that the corresponding angles are congruent. Sketch a counterexample. Explain your counterexample.
- 23. Critique Reasoning** Kelly was asked to determine whether $\triangle KLN$ is congruent to $\triangle MNL$. She noted that $\overline{KL} \cong \overline{MN}$, $\overline{KN} \cong \overline{ML}$, and that the three pairs of corresponding angles are congruent. She said that this is only five pairs of congruent corresponding parts, so it is not possible to conclude that $\triangle KLN$ is congruent to $\triangle MNL$. Do you agree? Explain.



- 24. Analyze Relationships** David uses software to draw two triangles. He finds that he can use a rotation and a reflection to map one triangle onto the other, and he finds that the image of vertex D is vertex L , the image of vertex V is vertex C , and the image of vertex W is vertex Y . In how many different ways can David write a congruence statement for the triangles? Explain.

Lesson Performance Task

For Kenny's science project, he is studying whether honeybees favor one color of eight-petal flowers over other colors. For his display, he is making eight-petal flowers from paper in various colors. For each flower, he'll cut out eight triangles like the one in the figure.



- Find x , the measure in degrees of the top angle of each triangle. Explain how you found x .
- Find y , the measure in degrees of the two base angles of each triangle. Explain how you found y .
- Explain how Kenny could confirm that one of his triangles is congruent to the other seven.