21.1 Solving Equations by Factoring $x^2 + bx + c$

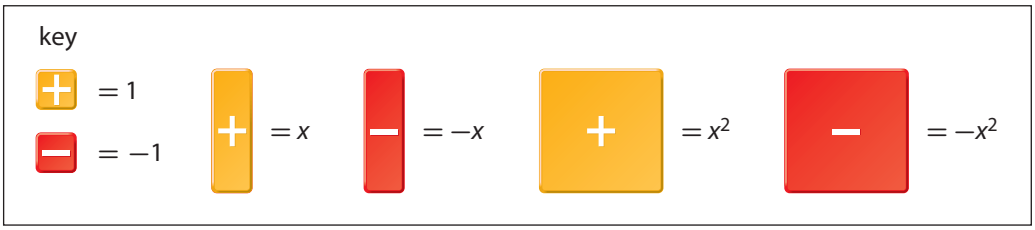
Resource Locker

Essential Question: How can you use factoring to solve quadratic equations in standard form for which $a = 1$?

Explore 1 Using Algebra Tiles to Factor $x^2 + bx + c$

In this lesson, multiplying binomials using the FOIL process will be reversed and trinomials will be factored into two binomials. To learn how to factor, let's start with the expression $x^2 + 7x + 6$.

(A) Identify and draw the tiles needed to model the expression $x^2 + 7x + 6$.



The tiles needed to model the expression $x^2 + 7x + 6$ are:

? x^2-tiles(s), ? x-tile(s), and ? unit tile(s).

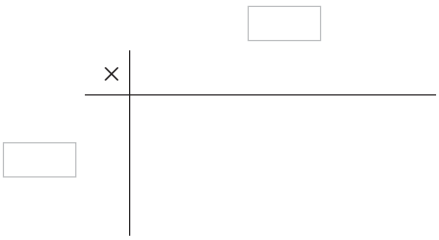
(B) Arrange and draw the algebra tiles on a grid. Place the ? x^2-tile(s) in the upper left corner and arrange the ? unit tiles in two rows and three columns in the lower right corner.

(C) Try to complete the rectangle with the x-tiles. Notice that only ? x-tiles fit on the grid, which leaves out ? tile(s), so this arrangement is not correct.

(D) Rearrange the unit tiles so that all of the ? x-tiles fit on the mat.

(E) Complete the multiplication grid by placing the factor tiles on the sides. Then write the factors modeled in this product.

$x^2 + 7x + 6 = (x + ?)(x + ?)$



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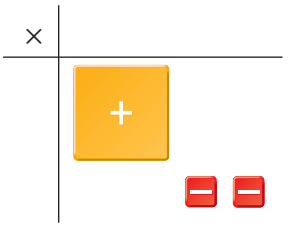
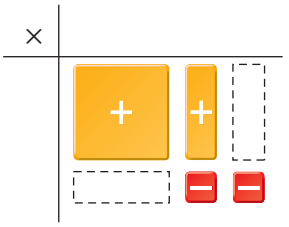
F Now let's look at how to factor a quadratic expression with a negative constant term. Use algebra tiles to factor $x^2 + x - 2$. Identify the tiles needed to model the expression.

? positive x^2-tile(s), ? positive x-tile(s), and ? negative unit tile(s)

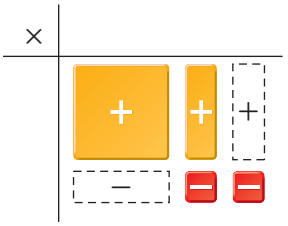
G Arrange the algebra tiles on the grid. Place the ? positive x^2-tile(*s*) in the upper left corner and arrange the ? negative unit tiles in the lower right corner.

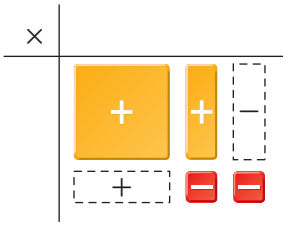


H Try to fill in the empty spaces on the grid with x-tiles. There is/are ? positive x-tile(*s*) to place on the grid, so there will be ? empty places for x-tiles.



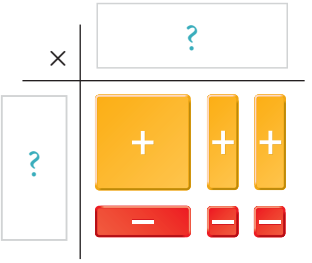
I Complete the rectangle on the mat by using *zero pairs*. Add ? positive x-tile(*s*) and ? negative x-tile(s) to the grid in such a way that the factors work with all the tiles on the mat. Which mat shows the correct position of zero pairs?





J Complete the multiplication grid by placing the factor tiles on the sides. Then write the factors modeled in this product.

$$x^2 + x - 2 = \left(x + \boxed{?}\right)\left(x - \boxed{?}\right)$$



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Reflect

1. Are there any other ways to factor the polynomial $x^2 + 7x + 6$ besides $(x + 6)(x + 1)$? Explain.

2. **Discussion** If c is positive in $x^2 + bx + c$, what sign can the constant terms of the factors have? What about when c is negative?

Explore 2 Factoring $x^2 + bx + c$

To factor $x^2 + bx + c$, you need to find two factors of c whose sum is b.

Factoring $x^2 + bx + c$	
WORDS	**EXAMPLE**
To factor a quadratic trinomial of the form $x^2 + bx + c$, find two factors of c whose sum is b. If no such Integers exist, the trinomial is not factorable.	To factor $x^2 + 9x + 18$, look for factors of 18 whose sum is 9. Factors of 18 — Sum 1 and 18 — 19 — x 2 and 9 — 11 — x 3 and 6 — 9 — ✓ $x^2 + 9x + 18 = (x + 3)(x + 6)$

If c is positive, the constant terms of the factors have the same sign.

If c is negative, then one constant term of the factors is positive and one is negative.

(A) First, look at $x^2 + 11x + 30$. Find the values of b and c. $b =$? $c =$?

(B) The sign of c is ?. The sign of the factors will be the ?.

(C) List the factor pairs of c, 30, and find the sum of each pair.

Factors of 30	Sum of Factors
1 and ?	1 + ? = ?
2 and ?	2 + ? = ?
3 and ?	3 + ? = ?
5 and ?	5 + ? = ?

(D) The factor pair whose sum equals b is ?.

Use this factor pair to factor the polynomial. $x^2 + 11x + 30 = (x + ?)(x + ?)$

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(E) Now, look at $x^2 + 13x - 30$. Find the values of b and c.

$b =$? $\quad c =$?

(F) The sign of c is ? . The sign of the factors will be the ? .

(G) List the factor pairs of c, –30, and find the sum of each pair.

Factors of −30	Sum of Factors
1 and ?	1 + ? = ?
2 and ?	2 + ? = ?
3 and ?	3 + ? = ?
5 and ?	5 + ? = ?
−1 and ?	−1 + ? = ?
−2 and ?	−2 + ? = ?
−3 and ?	−3 + ? = ?
−5 and ?	−5 + ? = ?

(H) The factor pair whose sum equals b is ? .

Use this factor pair to factor the polynomial.

$x^2 + 13x - 30 = (x + ?)(x - ?)$

Reflect

3. **Discussion** When factoring a trinomial of the form $x^2 + bx + c$, where c is negative, one binomial factor contains a positive factor of c and one contains a negative factor of c. How do you know which factor of c should be positive and which should be negative?

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Explain 1 Solving Equations of the Form $x^2 + bx + c = 0$ by Factoring

As you have learned, the Zero Product Property can be used to solve quadratic equations in factored form.

Example 1 **Solve each equation by factoring. Check your answer by graphing.**

(A) $x^2 - 8x = -12$

First, write the equation in the form $x^2 - bx + c = 0$.

$x^2 - 8x = -12$ Original equation

$x^2 - 8x + 12 = 0$ Add 12 to both sides.

The expression $x^2 - 8x + 12$ is in the form $ax^2 + bx + c$, with $b < 0$ and $c > 0$, so the factors will have the same sign and they both will be negative.

Factors of 12	Sum of Factors
-1 and -12	$-1 + (-12) = -13$
-2 and -6	$-2 + (-6) = -8$
-3 and -4	$-3 + (-4) = -7$

The factor pair whose sum equals -8 is -2 and -6. Factor the equation, and use the Zero Product Property.

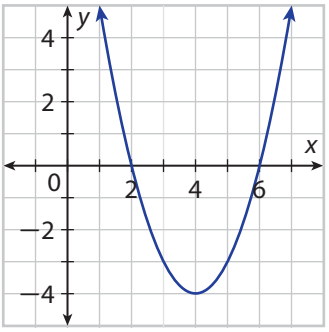
$$x^2 - 8x + 12 = 0$$

$$(x - 2)(x - 6) = 0$$

$$x - 2 = 0 \quad \text{or} \quad x - 6 = 0$$

$$x = 2 \qquad\qquad x = 6$$

The zeros of the equation are 2 and 6. Check this by graphing the related function, $f(x) = x^2 - 8x + 12$.



The x-intercepts of the graph are 2 and 6, which are the same as the zeros of the equation. The solutions of the equation are 2 and 6.

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(B) $x^2 - 2x = 15$

First, rewrite the expression in the form $x^2 + bx + c = 0$.

$x^2 - 2x = 15$	Original equation
$x^2 - 2x - \boxed{15} = 0$	Subtract 15 from both sides.

To find the zeros of the equation, start by factoring. List the factor pairs of c and find the sum of each pair. Since $c < 0$, the factors will have opposite signs. Since $c < 0$ and $b < 0$, the factor with the greater absolute value will be negative.

Factors of −15	Sum of Factors
1 and $\boxed{-15}$	$1 + \boxed{-15} = \boxed{-14}$
3 and $\boxed{-5}$	$3 + \boxed{-5} = \boxed{-2}$
−1 and $\boxed{15}$	$-1 + \boxed{15} = \boxed{14}$
−3 and $\boxed{5}$	$-3 + \boxed{5} = \boxed{2}$

The factor pair whose sum equals −2 is 3 and −5. Factor the equation, and use the Zero Product Property.

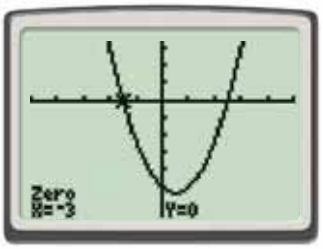
$$x^2 - 2x - 15 = 0$$

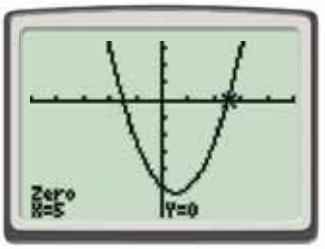
$$\left(x + \boxed{3}\right)\left(x - \boxed{5}\right) = 0$$

$$x + 3 = 0 \quad \text{or} \quad x - 5 = 0$$

$$x = \boxed{-3} \qquad x = \boxed{5}$$

The zeros of the equation are $\boxed{-3}$ and $\boxed{5}$. Check this by graphing the related function, $f(x) = x^2 - 2x = 15$.





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Your Turn

Solve each equation.

4. $x^2 + 15x = -54$

5. $x^2 - 13x = -12$

6. $x^2 - x = 56$

Explain 2 Solving Equation Models of the Form $x^2 + bx + c = 0$ by Factoring

Some real-world problems can be solved by factoring a quadratic equation.

Example 2 **Solve each model by factoring.**

Architecture A rectangular porch has dimensions of $(x + 12)$ and $(x + 5)$ feet. If the area of the porch floor is 120 square feet, what are its length and width?

Write an equation for the problem. Substitute 120 for A for the area of the porch.

$$(x + 12)(x + 5) = A$$

$$x^2 + 17x + 60 = A$$

$$x^2 + 17 + 60 = 120$$

$$x^2 + 17x - 60 = 0$$

The factors of -60 that have a sum of 17 are 20 and -3. Use Zero Product Property to find x.

$$(x + 20)(x - 3) = 0$$

$$x + 20 = 0 \quad \text{or} \quad x - 3 = 0$$

$$x = -20 \qquad x = 3$$

Since the area cannot be negative, $x = 3$ feet.

Therefore, the dimensions of the porch are $3 + 12 = 15$ feet long and $3 + 5 = 8$ feet wide.

Elaborate

7. How are the solutions of a quadratic equation related to the zeros of the related function?

8. **Essential Question Check-In** How can you solve a quadratic equation by factoring?

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Use algebra tiles to model the factors of each expression.

1. $x^2 + 6x + 8$

$x^2 + 6x + 8 = (x \boxed{?})(x \boxed{?})$

2. $x^2 + 2x - 3$

$x^2 + 2x - 3 = (x \boxed{?})(x \boxed{?})$

Factor the expressions.

3. $x^2 - 15x + 44$

4. $x^2 + 22x + 120$

5. $x^2 + 14x - 32$

6. $x^2 - 12x - 45$

7. $x^2 + 10x + 24$

8. $x^2 + 7x - 8$

Solve each equation.

9. $x^2 + 19x = -84$

10. $x^2 - 18x = -56$

11. $x^2 - 12x + 27 = 0$

12. $x^2 - 9x - 10 = 0$

13. $x^2 + 6x = 135$

14. $x^2 + 13x = -40$

15. $x^2 + x - 132 = 0$

16. $x^2 - 14x = 32$

17. **Construction** The area of a rectangular fountain is $(x^2 + 12x + 20)$ square feet. A 2-foot walkway is built around the fountain. Find the dimensions of the outside border of the walkway.

18. The area of a room is 396 square feet. The length is $(x + 3)$, and the width is $(x + 7)$ feet. Find the dimensions of the room.

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19. A rectangular Persian carpet has an area of $(x^2 + x - 20)$ square feet and a length of $(x + 5)$ feet. The Persian carpet is displayed on a wall. The wall has a width of $(x + 2)$ feet and an area of $(x^2 + 17x + 30)$ square feet. Find the dimensions of the rug and the wall if $x = 20$ feet.

20. The area of a poster board is $x^2 + 3x - 10$ square inches. Find the dimensions of the poster board if $x = 14$.

21. Match the equation to its solutions.

A. $x^2 - 3x - 18 = 0$	**a.** ___?___	3 and 6
B. $x^2 - 9x + 18 = 0$	**b.** ___?___	-3 and -6
C. $x^2 + 3x - 18 = 0$	**c.** ___?___	3 and -6
D. $x^2 + 9x + 18 = 0$	**d.** ___?___	-3 and 6

H.O.T. Focus on Higher Order Thinking

22. **Explain the Error** Amelie found the solutions of the equation $x^2 - x = 42$ to be 6 and -7. Explain why this answer is incorrect. Then, find the correct solutions.

23. **Communicate Mathematical Ideas** Rico says the expression $x^2 + bx + c$ is factorable when $b = c = 4$. Are there any other values where $b = c$ that make the expression factorable? Explain.

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24. **Multi-Step** A homeowner wants to enlarge a rectangular closet that has an area of $(x^2 + 3x + 2)$ square feet. The length of the closet is greater than the width. After construction, the area will be $(x^2 + 8x + 15)$ square feet.

a. Find the dimensions of the closet before construction.

b. Find the dimensions of the closet after construction.

c. By how many feet will the length and width increase after construction?

25. **Critical Thinking** Given $x^2 + bx + 64$, find all the values of b for which the quadratic expression has factors $(x + p)$ and $(x + q)$, where p and q are integers.

Lesson Performance Task

Part of the roof of a factory is devoted to mechanical support and part to green space. The area of the roof R of a large building can be modeled by the polynomial $2x^2 - 251x + 80{,}000$ and the area M that is devoted to mechanical support can be modeled by the polynomial $x^2 + 224x + 31{,}250$. Given that the area G of the green space is 123,750 square feet, write and solve quadratic equations to find the dimensions of the green space.

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